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minus x; and the ability to get an equation of which the square can be completed depends upon this operation,

In the seventh solution Dr. Zerr lets $t=3-y$ and gets $x^2-4=t$ and $x-2=t(3-y)$. Now the only integral value of t that will satisfy this assumption is zero; and this gives $x=2$ and $y=3$. And the the rest of the work is only necessary to show a little algebraic skill in getting a form the square of which can be completed. We are to observe, too, that when the square root is taken the double sign must be excluded.

In regard to the eleventh solution I ask how the solver got the $10ab=40+10b$ to subtract from (5)? I think he will admit that he found it by trial; or if not, he knew the values sought and from them found the member by which (4) must be multiplied to get this result. In every similar example this number is $2(y^2-x^2)(y>x)$. In the present example $2(9-4)=10$. Had the example been $\left\{\begin{matrix} x^2+y=9 \\ x+y^2=27 \end{matrix}\right\}$ when $x=2$ and $y=5$, the number with which (4) must be multiplied is $2(25-4)=42$. So by knowing the roots in tnis mode of solution we always can find what to subtract to complete the square. But if we do not know the roots sought the finding of this quantity is a mere work *trial*; and would require much more labor than to find the values by inspection.

So I have come to the conclusion that when there are integral values for x and y they are best found by inspection, and the other values are best found by the use of Sturm's Theorem and Horner's Method of approximation.

M. C. STEVENS.

Purdue University.

This problem, with four different solutions (all of which have been published in the MONTHLY) may be found in the *School Visitor*, Vol. III., pp. 114-115, and the editor, Mr. John S. Royer, calls it the "Yale Problem." Three solutions of this problem may also be found in *The Mathematical Visitor*, Vol. II., p. 3. and two solutions of it appeared in Vol. II., p. 25, of *The Analyst*.

$x^2+y=a$ is the equation of a parabola whose axis coincides with the Y-axis of reference, and its infinite branches extending in the negative direction; $x+y^2=b$ is the equation of a parabola whose axis coincides with the X-axis of reference and its infinite branches extending in the negative direction. These two curves may intersect in four points, intersect in two points and touch in one, intersect in two points, touch in one point or not intersect at all. The equations considered as simultaneous may, therefore, have four real roots, all different; four real roots, two equal and two different; two real roots, equal or different, and two imaginary roots; or four imaginary roots. The solution of the general case leads to a biquadratic which cannot be solved by quadratics.

EDITOR F.]

GEOMETRY.

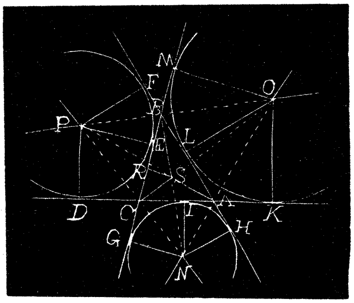
108. Proposed by **NELSON L. RORAY**, Bridgeton, N. J.

ABC is a triangle. O_1, O_2, O_3 centers of escribed circles. Prove altitudes of triangle $O_1O_2O_3$ are concurrent at center of incribed circle.

I. Solution by **M. A. GRUBER, A. M.**, War Department, Washington, D. C.

Take the figure of my solution of Problem 97, Vol. V, No. 10, page 231, THE AMERICAN MATHEMATICAL MONTHLY.

We are to prove BN, AP, and CO the respective altitudes of $\triangle PON$, and passing through S, the center of inscribed circle of $\triangle ABC$.



$\triangle BGN$ and $\triangle BHN$ are equal; for $NG=NH$ (radii of same circle); $\angle BGN=\angle BHN$ $=$right triangle (radius to point of tangency), and BN is common.

$\therefore$ $\angle ABN=\angle CBN$; and as BN bisects $\angle ABC$, BN passes through S, the center of inscribed circle.

$\angle ABO+\angle ABN+\angle CBN+\angle CBP=$ two right triangles. But $\angle ABO=\angle CBP$ (property of escribed circles), and $\angle ABN=\angle CBN$. $\therefore$ $2\angle ABO=2\angle ABN=$two right angles. Whence $\angle ABO+\angle ABN=\angle NBO=$right triangle.

$\therefore$ BN is perpendicular to PO.

$\therefore$ BN is the altitude of $\triangle PON$ let fall from the vertex N, and passes through the center of the inscribed circle of $\triangle ABC$.

In a similar manner, AP and CO can be proved the other two altitudes passing through S.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.**

ABC is the pedal triangle of $O_1O_2O_3$.

The altitudes of $O_1O_2O_3$ bisect the angles of its pedal triangle.

$\therefore$ The altitudes are concurrent at the in-center of ABC.

III. **Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.**

Using trilinears, and ABC as the triangle of reference, the equation to BC, CA, AB being $\alpha=0\ldots\ldots(1)$, $\beta=0\ldots\ldots(2)$, $\gamma=0\ldots\ldots(3)$, respectively, and noticing that O_1O_2, O_2O_3, O_3O_1 are the external bisectors of angles C, A, B, respectively, we have their equations, in order,

$$\alpha+\beta=0\ldots\ldots(4),\ \beta+\gamma=0\ldots\ldots(5);\ \gamma+\alpha=0\ldots\ldots(6).$$

Any line through the intersection of (4) and (5) is given by $\alpha+\beta+k(\beta+\gamma)=0$, or $(1+k)\alpha+\beta+k\gamma=0\ldots\ldots(7)$.

The condition that (7) is perpendicular to (6) is

$$(1+k)(1-\cos A-\cos B-\cos C)=0\ldots\ldots(8).$$

This in general requires that $k=-1\ldots\ldots(9)$, and (7) is $\beta-\gamma=0\ldots\ldots(10)$, the internal bisector of $\angle C$. This really proves the theorem.

IV. **Solution by P. H. PHILBRICK, C. E., Lake Charles, La.; G. I. HOPKINS, A. M., Professor of Mathematics and Physics, High School, Manchester, N. H.; W. H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; ALOIS F. KOVARIK, Professor of Mathematics, Decorah Institute, Decorah, Ia.; MELVIN ENGER, Decorah, Ia.; P. S. BERG, B. Sc., Principal of Schools, Larimore, N. D.; and J. O. MAHONEY, B. E., M. Sc., Master of Mathematics and Science, Carthage Graded and High School, Carthage, Tex.**

In the figure of the first solution, change N to O_2, P to O_1, O to O_3, and S to O.

Since COO_3 bisects the angle C, it passes through the centers O of the inscribed circle and O_3 of the escribed circle. Moreover, since CO_1 bisects the supplement of C, O_1CO_3 is a right angle. Hence CO_3 is an altitude of $O_1O_2O_3$. For the same reason, AO_1 as well as BO_2 pass through O and are altitudes of $O_1O_2O_3$.

Excellent demonstrations of this proposition were also furnished by *COOPER D. SCHMITT, E. D. SCALES, J. SCHEFFER,* and *CHAS. C. CROSS.* Mr. Cross sent in two different demonstrations.